

Chapter 4: Primitives and Integrals

- **Integrations by parts.**
- We have $(UV)' = U'V + V'U$
- $\Leftrightarrow UV' = (UV)' - U'V$
- $\Leftrightarrow \int UV' dx = \int [(UV)' - U'V] dx$
- $\Leftrightarrow \int UV' dx = \int (UV)' dx - \int U'V dx$
- $\Leftrightarrow \int UV' dx = \int (UV)' dx - \int U'V dx$
- **Example :** Determine $\int x e^x dx$

$$U = x$$

$$U' = 1$$

$$V' = e^x$$

$$V = \int e^x dx = e^x$$

$$I = x e^x - \int 1 \cdot e^x dx = x e^x - e^x + C$$

$$(UV)' = U'V + V'U$$

$$UV' = (UV)' - U'V$$

$$\int UV' dx = \int (UV)' dx - \int U'V dx$$

$$\boxed{\int UV' dx = UV - \int U'V dx}$$

Chapitre 4 : Primitives et intégrales

• *In the case of definite integrals. The formula will be:*

• $\Leftrightarrow \int_a^b UV' dx = UV \Big|_a^b - \int_a^b U'V dx$

• **Example :** Determine $\int_1^e \ln(x) dx$

$$U = \ln x ; \quad v' = 1$$

$$U' = \frac{1}{x} ; \quad V = \int 1 dx = x$$

$$I = \left[x \ln x \right]_1^e - \int_1^e \frac{1}{x} \cdot x dx = \left[x \ln x \right]_1^e - \int_1^e 1 dx$$

$$= \left(e \ln e - 1 \ln 1 \right) - \left(x \right)_1^e$$

$$= e - 0 - (e - 1) = e - e + 1 = 1$$

• a. We have, $\int \frac{x}{(x+1)^3} dx = ??$ $\int x (x+1)^{-3} dx$ $u = x+1 \rightarrow u' = 1$

$$u = x$$

$$u' = 1$$

$$v' = (x+1)^{-3}$$

$$v = \int (x+1)^{-3} dx$$

$$u = x+1 \rightarrow u' = 1$$

$$\int u' u^3 dx = \frac{(x+1)^{-3+1}}{-3+1} = -\frac{1}{2} (x+1)^{-2}$$

$$I = x \left[-\frac{1}{2} (x+1)^{-2} \right] - \int 1 \cdot \left[-\frac{1}{2} (x+1)^{-2} \right] dx = -\frac{1}{2} \frac{x}{(x+1)^2} + \frac{1}{2} \int (x+1)^{-2} dx$$

$$= -\frac{1}{2} \frac{x}{(x+1)^2} + \frac{1}{2} \frac{(x+1)^{-2+1}}{-2+1} + C = -\frac{1}{2} \frac{x}{(x+1)^2} - \frac{1}{2} \times \frac{1}{x+1} + C$$

• **b.** We have, $I = \int x\sqrt{1+x}dx = ??$ $I = \int x \cdot (1+x)^{\frac{1}{2}} dx$ $u = 1+x$ ~~$u' = 1$~~

$$u = x \quad ; \quad v' = (1+x)^{\frac{1}{2}}$$

$$u' = 1$$

$$v = \int (1+x)^{\frac{1}{2}} = \frac{(1+x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{(1+x)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \cdot (1+x)^{\frac{3}{2}}$$

$u = 1+x \rightarrow u' = 1$

$$\int u' u^n dx = \frac{u^{n+1}}{n+1}$$

$$I = \frac{2}{3} x (1+x)^{\frac{3}{2}} - \frac{2}{3} \int 1 \cdot (1+x)^{\frac{3}{2}} dx = \frac{2}{3} x (1+x)^{\frac{3}{2}} - \frac{2}{3} \frac{(1+x)^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$u = 1+x$
 $u' = 1$

$$= \frac{2}{3} x (1+x)^{\frac{3}{2}} - \frac{2}{3} \frac{(1+x)^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3} x (1+x)^{\frac{3}{2}} - \frac{4}{15} (1+x)^{\frac{5}{2}} + C = \frac{2}{3} x \sqrt{(1+x)^3} - \frac{4}{15} \sqrt{(1+x)^5} + C$$